Mathematical language

Mathematics has a language all of its own – not just its own vocabulary but the way sentences and maths questions are structured. Pupils who struggle with language, literacy or maths will make better progress if teachers can be explicit about how maths language and literacy works.

> "A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the "***mathematics register***", in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes."
>
> **M.A.K Halliday 1967**

There are several literacy skills areas related to understanding mathematics:

- Speaking and listening in the mathematics classroom, also called talk for learning.
- Specialised mathematics vocabulary (tier three) often derived from Latin or Greek roots (e.g., *isosceles, congruent, polygon, quadrilateral)*.
- Familiar everyday words which have specific mathematical meanings (also tier three, e.g., *power, root, odd, face*).
- Reading mathematical discourse – understanding the way paragraphs and sentences are organised, particularly in word problems.
- Writing mathematically – setting out question workings and writing explanations.

Sentence structures

Here's a typical mathematical sentence which illustrates mathematical **generalising**.

*"The angles of a triangle **add up** to 180 degrees,"*

It uses plural nouns and present tense verb.

Another typical type of mathematical discourse might be **comparing** two objects.

*"This triangle is **twice as large as** that one."*

The sentence contains a comparative phrase *'as … (adjective) … as'.*

A further type of discourse would be used for **describing position,** transformations and reflection of shapes.

Can you list ten typical sentence patterns pupils need to understand?

Speaking and listening to develop understanding

Changes to mathematics qualifications now require pupils more than ever to be able to work on problems involving more than one area of mathematics. They are required to explain, reason and solve problems where conceptual understanding is assessed rather than their ability to apply a simple procedure. Collaborative learning is a very important tool in developing concepts and in enabling teachers to assess pupils' understanding and identify misconceptions. This requires a good deal of active discussion and opportunities to think out loud with pupils learning from each other.

Discussion and dialogue needs to be managed effectively in order to get the most out of it. It is important that pupils are given thinking time, opportunities to talk out loud, and get some feedback on their ideas and reasons before writing them down. It is particularly important for EAL learners and those from disadvantaged groups to have the chance to discuss difficult concepts informally, before moving towards formal written methods.

The following strategies all support talk for learning:

1. Group problem solving
2. Follow me lesson starters (see appendix)
3. True or false discussion (see appendix)
4. Think, pair, share

Expanding mathematical vocabulary

Effective maths teachers...

- Will be aware of the language demands of particular tasks and how specialist words are used within a specific topic.
- Model and revisit new mathematical language, using mnemonics and visual displays.
- Explicitly teach specialist vocabulary in context and ensure that pupils have opportunities to pronounce, explore and practise using new vocabulary.
- Will expect pupils to use correct mathematical terms and notation while exploring their insights.

Specialised words

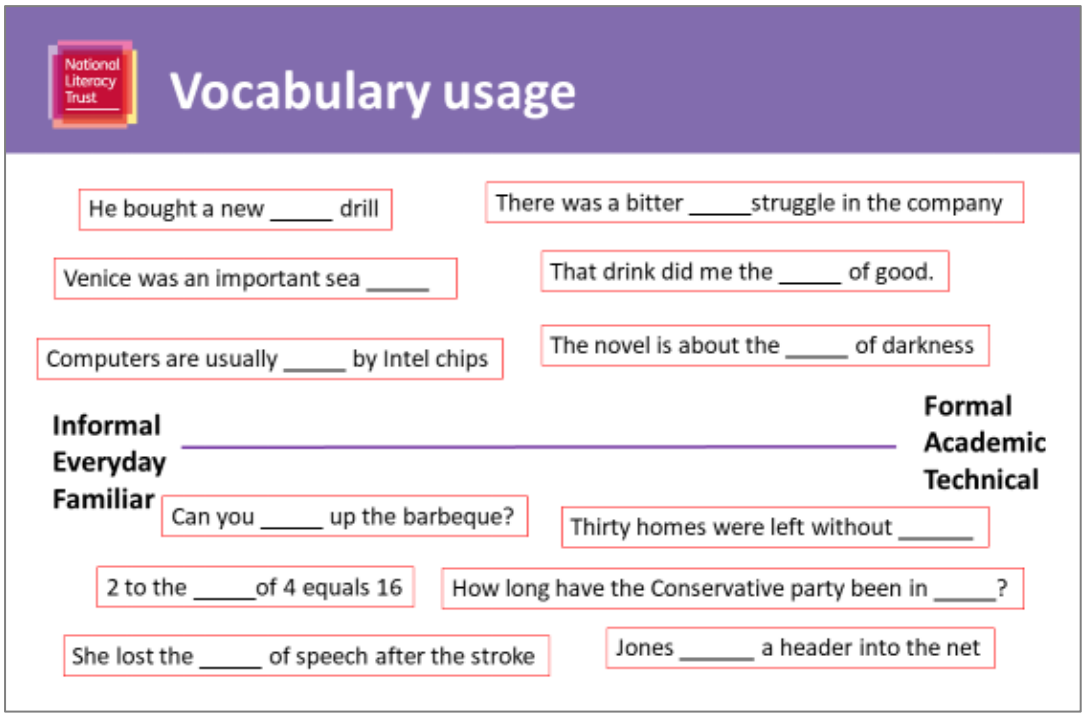
Make sure pupils can use morphology strategies to break down unfamiliar, specialist words: Explain key prefixes, for example Latin and Greek number prefixes (see the number prefix

sheet that supports this resource (visit https://literacytrust.org.uk/resources/developing-literacy-in-secondary-mathematics/).

Prefix	Root	Suffix
circum	fer	ence
dia	gon	al
re	flec	tion

Tricky words with an everyday meaning and a specialised meaning

Be explicit when you introduce these word to pupils. The word **'power'** has many meanings in many contexts. You might use a lesson starter like the one below to help them understand that context is everything!
Find the missing word and then sequence the sentences from most familiar to least.



Reading word problems

Longer, multi-step word problems can cause comprehension difficulties at all ages and stages. Research suggests (Barwell 2011) that pupils treat word problems too realistically and suspend their sense-making abilities. They try to make sense of them as narrative. EAL learners and those from disadvantaged backgrounds tend to have greater difficulties due to the unfamiliar contexts.

However, when writing their *own* word problems, students are able to 'mathematise' real world scenarios. The key to success is to draw on this ability so that students respond to the form and structure of unfamiliar word problems.

Modelling

To support student understanding, teachers need to explicitly demonstrate how a typical maths word problem is structured by using a visualizer and/or reading aloud. (See box below.) Pupils will benefit from working collaboratively to understand complex problems. You might assign roles to ensure balanced participation and use a reading the question note-making framework to guide them. Afterwards, ask pupil to write their own versions of common question types.

Generic three-part question structure

The information is arbitrary in relation to the scenario (x buys a y… its price is z).
There is an ambiguous use of verb tenses.
(For example, **present tense** is used for facts rather than past tense used in stories.)

1. **Scenario**

In a sale, the ratio of the sale price to the normal price **is** 3:5.

2. **Information**

Selina buys **a jacket** in the sale. Its normal **price is £45.**

3. **Question**

What is the sale price of the jacket?

Pupils need to focus on the facts not the story. Selina herself is irrelevant.

Alternatively, try giving students two similar problems to compare and contrast rather than answer. This way pupils focus on the structure of the question and how it works before trying to solve it.

Written communication

Composing a coherent answer may be an even bigger challenge than reading the question as there is a significant barrier for pupils in writing what might be called a mathematical paragraph. Once pupils have identified the mathematics they need to use, completed their calculations and found an answer they tend to be quite happy that the job is done, whatever it looks like on the page.

With complex problems it can be hard to follow pupils' working out and give them full credit for their solution, particularly if they have made one or two errors along the way. Teachers may need to explore how much importance to place on the quality of pupils' written

communication and the impact that this can have. We need to emphasise that the solution is only a part of the answer in the same way that a conclusion is only a part of an essay.

Reasoning and explaining in writing also tends to cause problems as often pupils say, *' I know why but I don't know how to write it down.'* Craig (2011), says using writing to document approaches to problem solving, such as *devising a plan, carrying out the plan, and looking back*, promotes metacognitive thinking and a deeper understanding of concepts.

Modelling writing in mathematics

Modelling involves the teacher demonstrating the writing process while "thinking aloud". It is used to make explicit to students the skills, decisions and processes that are normally hidden. As teachers model, they can demonstrate alterations and revise and edit information thus encouraging students to do the same.

When modelling:

- Practise with a colleague before trying in front of a class as it can feel strange at first. You may find it helps to write yourself a 'script' or note down key words and phrases.
- Explain to students that they should just listen and that you will answer any questions yourself.
- Read the problem aloud, explaining your thought processes and the steps you would take to solve the problem yourself. Maybe number the steps as you go.
- Write record your workings as you talk and leave the model answer on the board for reference.
- Give students a similar problem, preferably using the same format with different values.
- Ask students work in pairs, reading the problem and discussing their answers and making notes following the model they have been shown.
- Then join the pairs to make fours, compare solutions and work together to refine and produce one "best answer" from the group.

Flow chart solutions

A simple flow chart can help pupils to think about steps in calculations and scaffold their solutions. The left-hand side of the flowchart is used for pupils to explain in words the stages of their solution. The right-hand side is where they write the relevant calculations. For differentiation, some of the left-hand sections can be pre-populated.

If you want pupils to write summaries of mathematical methods such as how to find the area of a triangle, a technique is to provide a pupils with a pre-written model within a staged teaching sequence. The example below demonstrates this. (From Access and Engagement in Mathematics 2002).

classroom example

Materials used to develop pupils' skills in writing about mathematics

Developing pupils' skills in writing about mathematics – a teaching sequence

Familiarisation
Look at examples of mathematics facts

Identifying features
Underline and label definitions, rules, examples, formulae

Teacher modelling
Writing on an OHT/board with contributions

Guided writing
Pupils writing with prompt sheet

Independent writing
Ready to try it alone!

Example of a teaching sequence

Writing about mathematics – prompts

- Write the title of the topic.
- Say what the topic is about (for example, 'Area').
- Explain any difficult mathematical words.
- Explain what you did, giving step-by-step instructions.
- Use diagrams.
- Try to write a mathematical rule or formula – for example:

  To work out the area of a rectangle you multiply the length by the width

  or: area = length × width

Example of a prompt sheet

More teaching ideas from Slough and Eton Church of England Business & Enterprise College

When faced with a words maths question, middle ability pupils at Slough and Eton would initially rush in to the problem, not reading the question properly and throw any maths they knew at it, hoping something was right. To encourage pupils to focus their attempts, teachers at Slough and Eton came up with some prompts to guide them.

Solving the problem

1. Highlight command word or phrase in yellow
2. Highlight specialist maths vocabulary in blue
3. Bullet point key information so you have it all in one place
4. Ask yourself what you are being asked to find/do?
5. Ask yourself what maths you have to do to answer the question – number instructions in order if more than one step
6. Do the maths, showing working step by step.

After completing the calculations, pupils were put into pairs to talk through what they did with a peer and explain their thinking using the following prompts.

Explanation

1. What did you need to do?
2. What maths did you use?
3. Explain why you did each step?
4. Are there any changes you would make next time?

2. Revisiting sample questions with the prompts in mind

Teachers took individual questions with four or more marks, and asked pupils to solve it using the prompts. Teachers found the simple act of highlighting key information forced pupils to read the question carefully, giving themselves time to think. This activity gave pupils a better idea of how to tackle longer questions methodically.

3. Challenging with questions

Teachers then supplemented the prompts by **asking questions** – such as, "If you divide, what is the answer telling you?"; "What word in the question tells you that information?"; "What have you worked out?"; "Is that what the question wanted?"

4. Using whiteboards

Instead of working in their books, **pupils work on mini white boards**. Once they have a method that works, they write it up in their book with an explanation. Teachers at Slough and Eton found this method worked well as pupils were more willing to try things when they were not sure. They did not like crossing out errors in their books.

5. Whole-class plenary

When this approach was introduced, teachers would ask a pupil to come up to the board and explain how they had got an answer by talking through their workings. As lessons continued, teachers found themselves asking fewer questions as pupils asked their own

instead and their explanations and confidence has improved dramatically.

Further reading and references

Barwell, R. 2011. What works? Connecting Language, Maths and Life. https://www.edugains.ca/resourcesLNS/Monographs/WhatWorksSeries/WW_Word_Problems.pdf Accessed 20.02.2023

Craig, T. S. 2011. Categorization and analysis of explanatory writing in mathematics. *International Journal of Mathematical Education in Science and Technology* 42 (7): 867–878.

Pimm, D. 1987 Speaking Mathematically: Communication in Maths classrooms. London, Routledge

Talking maths resources

True or false? Discuss these statements in pairs

1⁄3 is smaller than 3/6
3/4 is greater than 5/6
0.4 is equal to 40%
0.5 is the same as 4/8
15% of £20 is £1.50
12% of £10 is more than 10% of £12
2/3 is smaller than 75%
0.25 of 60 is equal to 50% of 30
2/8 is equal to 0.28

Follow me/loop game

Copy onto card and cut up. Give each pupil one card. (Some are easier to solve than others.) Each pupil reads their card aloud and whoever has the answer puts up hand and reads their own card… and so on until you get back to the start. You might like to time it.

I am 0.1 You are 2 tenths greater than 0.1	**I am 0.3** You are 9 hundredths greater than 0.3	**I am 0.39** You are 1 tenth smaller than 0.39	**I am 0.29** You are 3 tenths greater than 0.29
I am 0.59 You are 5 hundredths smaller than 0.59	**I am 0.54** You are 4 tenths greater	**I am 0.94** You are 4 hundredths smaller than 0.94	**I am 0.9** You are 2 tenths greater than 0.9
I am 1.1 You are 15 hundredths greater than 1.1	**I am 1.25** You are 3 hundredths smaller than 1.25	**I am 1.22** You are 4 tenths greater than 1.22	**I am 1.62** You are 1 tenth smaller than 1.62
I am 1.52 You are 8 hundredths greater than 1.52	**I am 1.60** You are 8 tenths smaller than1.6	**I am 0.80** You are 7 hundredths greater than 0.8	**I am 0.87** You are 4 hundredths smaller than 0.87
I am 0.83 You are 3 tenths smaller than 0.83	**I am 0.53** You are 8 hundredths greater than o.53	**I am 0.61** You are 1 tenth smaller than 0.61	**I am 0.51** You are 3 tenths smaller than 0.51
I am 0.21 You are 5 hundredths smaller than 0.21	**I am 0.16** You are 7 hundredths greater than 0.16	**I am 0.23** You are 9 hundredths smaller than 0.23	**I am 0.14** You are 4 hundredths smaller than 0.14